The release of the updated *FSA Test Item Specifications* is intended to provide greater specificity for item writers in developing items to be field tested in 2016. The revisions in the specifications will NOT affect the Spring 2015 Florida Standards Assessments. The enhanced explanations, clarifications, and sample items should assist item writers and other stakeholders in understanding the Florida Standards and the various types of test items that can be developed to measure student proficiency in the applicable content areas for 2016 and beyond.
The draft Florida Standards Assessments (FSA) Test Item Specifications (Specifications) are based upon the Florida Standards and the Florida Course Descriptions as provided in CPALMs. The Specifications are a resource that defines the content and format of the test and test items for item writers and reviewers. Each grade-level and course Specifications document indicates the alignment of items with the Florida Standards. It also serves to provide all stakeholders with information about the scope and function of the FSA.

Item Specifications Definitions

Also assesses refers to standard(s) closely related to the primary standard statement.

Clarification statements explain what students are expected to do when responding to the question.

Assessment limits define the range of content knowledge and degree of difficulty that should be assessed in the assessment items for the standard.

Item types describe the characteristics of the question.

Context defines types of stimulus materials that can be used in the assessment items.
Florida Standards Assessments

Modeling Cycle

The basic modeling cycle involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

Mathematical Practices:

The Mathematical Practices are a part of each course description for Grades 3-8, Algebra 1, Geometry, and Algebra 2. These practices are an important part of the curriculum. The Mathematical Practices will be assessed throughout.

<table>
<thead>
<tr>
<th>Mathematical Practices</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Make sense of problems and persevere in solving them.</strong></td>
<td>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</td>
</tr>
<tr>
<td><strong>Reason abstractly and quantitatively.</strong></td>
<td>Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent</td>
</tr>
<tr>
<td>MAFS.K12.MP.3.1: Construct viable arguments and critique the reasoning of others.</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td></td>
</tr>
<tr>
<td>Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MAFS.K12.MP.4.1: Model with mathematics.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the</td>
</tr>
</tbody>
</table>
situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MAFS.K12.MP.5.1:

Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
### MAFS.K12.MP.7.1:

**Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

### MAFS.K12.MP.8.1:

**Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
Technology-Enhanced Item Descriptions:

The Florida Standards Assessments (FSA) are composed of test items that include traditional multiple-choice items, items that require students to type or write a response, and technology-enhanced items (TEI). Technology-enhanced items are computer-delivered items that require students to interact with test content to select, construct, and/or support their answers.

Currently, there are nine types of TEIs that may appear on computer-based assessments for FSA Mathematics. For Grades 3 and 4 assessments, which will be paper-based tests in 2014-2015, and for students with an IEP or 504 plan that specifies a paper-based accommodation, TEIs will be modified or replaced with test items that can be scanned and scored electronically.

For samples of each of the item types described below, see the FSA Training Tests.

Technology-Enhanced Item Types – Mathematics

1. **Editing Task Choice** – The student clicks a highlighted word or phrase, which reveals a drop-down menu containing options for correcting an error as well as the highlighted word or phrase as it is shown in the sentence to indicate that no correction is needed. The student then selects the correct word or phrase from the drop-down menu. For paper-based assessments, the item is modified so that it can be scanned and scored electronically. The student fills in a circle to indicate the correct word or phrase.

2. **Editing Task** – The student clicks on a highlighted word or phrase that may be incorrect, which reveals a text box. The directions in the text box direct the student to replace the highlighted word or phrase with the correct word or phrase. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

3. **Hot Text** –
   a. **Selectable Hot Text** – Excerpted sentences from the text are presented in this item type. When the student hovers over certain words, phrases, or sentences, the options highlight. This indicates that the text is selectable (“hot”). The student can then click on an
option to select it. For paper-based assessments, a “selectable” hot text item is modified so that it can be scanned and scored electronically. In this version, the student fills in a circle to indicate a selection.

b. **Drag-and-Drop Hot Text** – Certain numbers, words, phrases, or sentences may be designated “draggable” in this item type. When the student hovers over these areas, the text highlights. The student can then click on the option, hold down the mouse button, and drag it to a graphic or other format. For paper-based assessments, drag-and-drop hot text items will be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

4. **Open Response** – The student uses the keyboard to enter a response into a text field. These items can usually be answered in a sentence or two. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

5. **Multiselect** – The student is directed to select all of the correct answers from among a number of options. These items are different from multiple-choice items, which allow the student to select only one correct answer. These items appear in the online and paper-based assessments.

6. **Graphic Response Item Display (GRID)** – The student selects numbers, words, phrases, or images and uses the drag-and-drop feature to place them into a graphic. This item type may also require the student to use the point, line, or arrow tools to create a response on a graph. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.

7. **Equation Editor** – The student is presented with a toolbar that includes a variety of mathematical symbols that can be used to create a response. Responses may be in the form of a number, variable, expression, or equation, as appropriate to the test item. For paper-based assessments, this item type may be replaced with a modified version of the item that can be scanned and scored electronically or replaced with another item type that assesses the same standard and can be scanned and scored electronically.

8. **Matching Item** – The student checks a box to indicate if information from a column header matches information from a row. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
9. **Table Item** – The student types numeric values into a given table. The student may complete the entire table or portions of the table depending on what is being asked. For paper-based assessments, this item type may be replaced with another item type that assesses the same standard and can be scanned and scored electronically.
Calculators:

General Designations:

**Calculator:**
Items only appear on Calculator Sessions.

**Calculator Neutral:**
Items appear on Calculator and No Calculator Sessions.

**No Calculator:**
Items only appear on No Calculator Sessions.

Types of Calculators:

**Grades 3–6**
- No calculator permitted for paper-based or computer-based tests.

**Grades 7–8**
- Online scientific calculator provided in the CBT platform for Sessions 2 and 3 of the Grades 7 and 8 FSA Mathematics tests.
- Online calculator may be accessed in the FSA Portal for use in the classroom.
- CBT students may request and use a handheld scientific calculator during Sessions 2 and 3. See below for a list of prohibited functionalities for handheld scientific calculators. Calculators that allow these prohibited functionalities may not be used.
- Students with paper-based accommodations must be provided a handheld scientific calculator for Sessions 2 and 3. See below for a list of prohibited functionalities for handheld scientific calculators. Calculators that allow these prohibited functionalities may not be used.

**End-of-Course (EOC)**
- Online scientific calculator provided in the CBT platform for Session 2 of the Algebra 1, Algebra 2, and Geometry tests.
- Online calculator may be accessed in the FSA Portal for use in the classroom.
- CBT students may request and use a handheld scientific calculator during Session 2. See below for a list of prohibited functionalities for handheld scientific calculators. Calculators that allow these prohibited functionalities may not be used.
- Students with paper-based accommodations must be provided a handheld scientific calculator for Session 2. See below for a list of prohibited functionalities for handheld scientific calculators. Calculators that allow these prohibited functionalities may not be used.
**Calculator Functionality:**

Students will need access to the following calculator functions:
- $\pi$
- $x^2$
- Square root ($\sqrt{}$)
- $x^3$ or $x^y$ for Grade 8 and EOCs
- $e^x$ for Algebra 1 and Algebra 2
- Trigonometric functions for Geometry and Algebra 2
- log and/or ln for Algebra 2

Students **may not** use a handheld calculator that has ANY of the following prohibited functionalities:
- CAS (an ability to solve algebraically) or a solver of any kind
- regression capabilities
- a table
- unit conversion other than conversions between degrees and radians (e.g., feet to inches)
- ability to simplify radicals
- graphing capability
- matrices
- a display of more than one line
- text-editing functionality (edit, copy, cut, and paste)
- the ability to perform operations with complex numbers
- the ability to perform prime factorization
- the ability to find gcd or lcm
- wireless or Bluetooth capability or Internet accessibility
- QWERTY keyboard or keypad
- need for an electrical outlet
- calculator peripherals
Reference Sheets:

- Reference sheets and z-tables will be available as online references (in a pop-up window). A paper version will be available for paper-based tests.
- Reference sheets with conversions will be provided for FSA Mathematics assessments in Grades 4–8 and EOC Mathematics assessments.
- There is no reference sheet for Grade 3.
- For Grades 4, 6, and 7, Geometry, and Algebra 2, some formulas will be provided on the reference sheet.
- For Grade 5 and Algebra 1, some formulas may be included with the test item if needed to meet the intent of the standard being assessed.
- For Grade 8, no formulas will be provided; however, conversions will be available on a reference sheet.
- For Algebra 2, a z-table will be available.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Conversions</th>
<th>Some Formulas</th>
<th>z-table</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>On Reference Sheet</td>
<td>On Reference Sheet</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>On Reference Sheet</td>
<td>With Item</td>
<td>No</td>
</tr>
<tr>
<td>6</td>
<td>On Reference Sheet</td>
<td>On Reference Sheet</td>
<td>No</td>
</tr>
<tr>
<td>7</td>
<td>On Reference Sheet</td>
<td>On Reference Sheet</td>
<td>No</td>
</tr>
<tr>
<td>8</td>
<td>On Reference Sheet</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Algebra 1</td>
<td>On Reference Sheet</td>
<td>With Item</td>
<td>No</td>
</tr>
<tr>
<td>Algebra 2</td>
<td>On Reference Sheet</td>
<td>On Reference Sheet</td>
<td>Yes</td>
</tr>
<tr>
<td>Geometry</td>
<td>On Reference Sheet</td>
<td>On Reference Sheet</td>
<td>No</td>
</tr>
</tbody>
</table>
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

<table>
<thead>
<tr>
<th>Item Types</th>
<th>Editing Task Choice – May require completing an informal argument on closure.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equation Editor – May require creating a value or an expression.</td>
</tr>
<tr>
<td></td>
<td>GRID – May require dragging and dropping expressions/statements to complete an informal argument.</td>
</tr>
<tr>
<td></td>
<td>Hot Text – May require dragging and dropping values/expressions to complete a polynomial.</td>
</tr>
<tr>
<td></td>
<td>Matching Item – May require matching equivalent polynomials.</td>
</tr>
<tr>
<td></td>
<td>Multiple Choice – May require selecting a value or an expression from a list.</td>
</tr>
<tr>
<td></td>
<td>Multiselect – May require selecting all equivalent expressions.</td>
</tr>
<tr>
<td></td>
<td>Open Response – May require creating a written explanation.</td>
</tr>
</tbody>
</table>

**Clarifications**

Students will relate the addition, subtraction, and multiplication of integers to the addition, subtraction, and multiplication of polynomials with integral coefficients through application of the distributive property.

Students will apply their understanding of closure to adding, subtracting, and multiplying polynomials with integral coefficients.

Students will add, subtract, and multiply polynomials with integral coefficients.

**Assessment Limits**

Items set in a real-world context should not result in a nonreal answer if the polynomial is used to solve for the unknown.

In items that require addition and subtraction, polynomials are limited to monomials, binomials, and trinomials. The simplified polynomial should contain no more than six terms.

Items requiring multiplication of polynomials are limited to a product of: two monomials, a monomial and a binomial, a monomial and a trinomial, two binomials, and a binomial and a trinomial.

**Stimulus Attributes**

Items may be set in a mathematical or real-world context.

Items may use function notation.

**Response Attributes**

Items may require the student to write the answer in standard form.

Items may require the student to recognize equivalent expressions.
Items may require the student to rewrite expressions with negative exponents, but items must not require the student to rewrite rational expression as seen in the standard MAFS.912.A-APR.4.7.

<table>
<thead>
<tr>
<th>Calculator</th>
<th>No</th>
</tr>
</thead>
</table>

**Sample Item**

Roxanne wants to test the idea that polynomials are closed under addition. Her work and explanation are shown.

Drag an equation and phrases to the boxes to complete the statement.

Add $3x^4 - 7x^3 + 12x - 9$

to $\phantom{3x^4 - 7x^3 + 12x - 9}$

This is closed because $\phantom{5x^2 - (2x)^3 + (4x)^{-1}}$ and the sum is a polynomial.

$5x^2 - (2x)^3 + (4x)^{-1}$

$-8x^{-2} - (3x)^3 - 12x^6 + 7$

the exponents

the variables

the coefficients

integers

real numbers

whole numbers
<table>
<thead>
<tr>
<th>MAFS.912.A-CED.1.1</th>
<th>Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational, absolute, and exponential functions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Also assesses</td>
<td>MAFS.912.A-REI.2.3</td>
</tr>
<tr>
<td>Also assesses</td>
<td>MAFS.912.A-CED.1.4</td>
</tr>
<tr>
<td></td>
<td>Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td>
</tr>
<tr>
<td></td>
<td>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law, V = IR, to highlight resistance, R.</td>
</tr>
</tbody>
</table>

**Item Types**

- **Editing Task Choice** – May require choosing a correct equation or the correct definition of a variable.
- **Equation Editor** – May require creating an equation, an inequality, or a value.
- **GRID** – May require dragging and dropping expressions/statements to complete a model.
- **Hot Text** – May require dragging and dropping values and/or expressions to create linear equations and inequalities or rearranging equations.
- **Multiple Choice** – May require identifying an equation, an inequality, or a value from a list of four choices.
- **Multiselect** – May require selecting an equation and identifying a variable.
- **Open Response** – May require creating a written explanation.

**Clarifications**

- Students will write an equation in one variable that represents a real-world context.
- Students will write an inequality in one variable that represents a real-world context.
- Students will solve a linear equation.
- Students will solve a linear inequality.
- Students will solve multi-variable formulas or literal equations for a specific variable.
- Students will solve formulas and equations with coefficients represented by letters.

**Assessment Limits**

In items that require the student to write an equation, equations are limited to exponential functions with one translation, linear functions, or quadratic functions.
Items may include equations or inequalities that contain variables on both sides.

In items that require the student to write an exponential function given ordered pairs, at least one pair of consecutive values must be given.

In items that require the student to write or solve an inequality, variables are restricted to an exponent of one.

Items that involve formulas should not include overused contexts such as Fahrenheit/Celsius or three-dimensional geometry formulas.

In items that require the student to solve literal equations and formulas, a linear term should be the term of interest.

Items should not require more than four procedural steps to isolate the variable of interest.

Items may require the student to recognize equivalent expressions but may not require a student to perform an algebraic operation outside the context of Algebra 1.

<table>
<thead>
<tr>
<th>Stimulus Attributes</th>
<th>Items assessing A-CED.1.1 and A-CED.1.4 must be placed in real-world context. Items assessing REI.2.3 do not have to be in a real-world context.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response Attributes</td>
<td>Items assessing REI.2.3 should not require the student to write the equation. Items may require the student to choose an appropriate level of accuracy. Items may require the student to choose and interpret units. For A-CED.1.1 and A-CED.1.4, items may require the student to apply the basic modeling cycle.</td>
</tr>
</tbody>
</table>

| Calculator | Neutral |
The table shows a company’s income and expenses over the last 7 days.

<table>
<thead>
<tr>
<th>Day of Week</th>
<th>Income</th>
<th>Expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun.</td>
<td>$169.56</td>
<td>$256.25</td>
</tr>
<tr>
<td>Mon.</td>
<td>$217.61</td>
<td>$195.79</td>
</tr>
<tr>
<td>Tues.</td>
<td>$150.89</td>
<td>$1208.55</td>
</tr>
<tr>
<td>Wed.</td>
<td>$409.73</td>
<td>$709.11</td>
</tr>
<tr>
<td>Thurs.</td>
<td>$687.45</td>
<td>$190.98</td>
</tr>
<tr>
<td>Fri.</td>
<td>$1563.09</td>
<td>$325.78</td>
</tr>
<tr>
<td>Sat.</td>
<td>$1267.92</td>
<td>$315.64</td>
</tr>
</tbody>
</table>

The company found that its weekly income and expenses were approximately the same from week to week.

A. Select the correct definition of the variable $x$.

B. Drag terms to the boxes and symbols to the circles to create an equation that can be solved to approximate the number of weeks it will take for the company’s income to be $10,000 more than its expenses.
<table>
<thead>
<tr>
<th>MAFS.912.A-CED.1.2</th>
<th>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Also assesses</td>
<td>Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</td>
</tr>
<tr>
<td>MAFS.912.A-REI.3.5</td>
<td>Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</td>
</tr>
<tr>
<td>Also assesses</td>
<td>Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td>
</tr>
<tr>
<td>MAFS.912.A-REI.3.6</td>
<td></td>
</tr>
<tr>
<td>Also assesses</td>
<td></td>
</tr>
</tbody>
</table>

**Item Types**

- **Editing Task Choice** – May require choosing the correct definition of a variable or completing an explanation or a proof.
  
  - **Equation Editor** – May require creating a set of equations, creating a set of inequalities, or giving an ordered pair.
  
  - **GRID** – May require graphing a representation of a set of equations, a set of inequalities, or an ordered pair; selecting a solution region; or dragging and dropping text to complete a proof.
  
  - **Hot Text** – May require selecting a solution or dragging and dropping text to complete a proof.

- **Multiple Choice** – May require identifying a set of equations, a set of inequalities, a value, an ordered pair, or a graph.

- **Multiselect** – May require identifying equations or inequalities.

- **Open Response** – May require writing an explanation.

**Clarifications**

- Students will identify the quantities in a real-world situation that should be represented by distinct variables.

- Students will write a system of equations given a real-world situation.

- Students will graph a system of equations that represents a real-world context using appropriate axis labels and scale.

- Students will solve systems of linear equations.

- Students will provide steps in an algebraic proof that shows one equation being replaced with another to find a solution for a system of equations.
| Students will identify systems whose solutions would be the same through examination of the coefficients. |
| Students will identify the graph that represents a linear inequality. |
| Students will graph a linear inequality. |
| Students will identify the solution set to a system of inequalities. |
| Students will identify ordered pairs that are in the solution set of a system of inequalities. |
| Students will graph the solution set to a system of inequalities. |

**Assessment Limits**

- Items that require the student to write a system of equations using a real-world context are limited to a system of 2 x 2 linear equations with integral coefficients if the equations are written in the form $Ax + By = C$.

- Items that require the student to solve a system of equations are limited to a system of 2 x 2 linear equations with integral coefficients if the equations are written in the form $Ax + By = C$.

- Items that require the student to graph a system of equations or inequalities to find the solution are limited to a 2 x 2 system.

- Items that require the student to write a system of inequalities using a real-world context are limited to integer coefficients.

**Stimulus Attributes**

- Items assessing A-CED.1.2 must be placed in a real-world context.

- Items assessing A-REI.3.5 must be placed in a mathematical context.

- Items assessing A-REI.3.6 and A-REI.4.12 may be set in a real-world or mathematical context.

- Items may result in infinitely many solutions or no solution.

**Response Attributes**

- Items may require the student to choose an appropriate level of accuracy.

- Items may require the student to choose and interpret the scale in a graph.

- Items may require the student to choose and interpret units.

- For A-CED.1.2, items may require the student to apply the basic modeling cycle.

**Calculator**

- Neutral
The system \( \frac{Px + Qy}{Fx + Gy} = \frac{R}{H} \) has the solution \((3, -1)\), where \(F, G, H, P, Q,\) and \(R\) are non-zero real numbers.

Select all the systems that are also guaranteed to have the solution \((3, -1)\).

- \( (P + F)x + (Q + G)y = R + H \)
  \( Fx + Gy = H \)

- \( (P + F)x + Qy = R + H \)
  \( Fx + (G + Q)y = H \)

- \( Px + Qy = R \)
  \( (3P + F)x + (3Q + G)y = 3H + R \)

- \( Px + Qy = R \)
  \( (F - 2P)x + (G - 2Q)y = H - 2R \)

- \( Px + Qy = R \)
  \( 5Fx + 5Gy = 5H \)
<table>
<thead>
<tr>
<th>MAFS.912.A-CED.1.3</th>
<th>Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item Types</td>
<td>Editing Task Choice – May require choosing a definition for a variable or a correct interpretation of a solution.</td>
</tr>
<tr>
<td></td>
<td>Equation Editor – May require creating a set of equations, inequalities, or values.</td>
</tr>
<tr>
<td></td>
<td>GRID – May require graphing a representation.</td>
</tr>
<tr>
<td></td>
<td>Hot Text – May require selecting a representation or dragging and dropping text to interpret solutions.</td>
</tr>
<tr>
<td></td>
<td>Multiple Choice – May require identifying an equation, an inequality, or a value.</td>
</tr>
<tr>
<td></td>
<td>Multiselect – May require selecting constraints, variable definitions, or equations that would model a context.</td>
</tr>
<tr>
<td></td>
<td>Open Response – May require writing an explanation.</td>
</tr>
<tr>
<td>Clarifications</td>
<td>Students will write constraints for a real-world context using equations, inequalities, a system of equations, or a system of inequalities.</td>
</tr>
<tr>
<td></td>
<td>Students will interpret the solution of a real-world context as viable or not viable.</td>
</tr>
<tr>
<td>Assessment Limits</td>
<td>In items that require the student to write an equation as a constraint, the equation may be a linear function.</td>
</tr>
<tr>
<td></td>
<td>In items that require the student to write a system of equations to represent a constraint, the system is limited to a 2 x 2 with integral coefficients.</td>
</tr>
<tr>
<td></td>
<td>In items that require the student to write a system of inequalities to represent a constraint, the system is limited to a 2 x 2 with integral coefficients.</td>
</tr>
<tr>
<td>Stimulus Attributes</td>
<td>Items must be set in a real-world context.</td>
</tr>
<tr>
<td></td>
<td>Items may use function notation.</td>
</tr>
<tr>
<td>Response Attributes</td>
<td>Items may require the student to choose an appropriate level of accuracy.</td>
</tr>
<tr>
<td></td>
<td>Items may require the student to choose and interpret the scale in a graph.</td>
</tr>
</tbody>
</table>
Items may require the student to choose and interpret units.

Items may require the student to apply the basic modeling cycle.

<table>
<thead>
<tr>
<th>Calculator</th>
<th>Neutral</th>
</tr>
</thead>
</table>

### Sample Item

<table>
<thead>
<tr>
<th>Item Type</th>
<th>Multiple Choice</th>
</tr>
</thead>
</table>

The production cost, \( C \), in thousands of dollars, for a toy company to manufacture a ball is given by the model \( C(x) = 75 + 21x - 0.72x^2 \), where \( x \) is the number of balls produced in one day, in thousands. The company wants to keep its production cost at or below $125,000. The graph shown models the situation.

![Graph showing production cost model](image)

What is a reasonable constraint for the model?

- \( A \) \(-3.2 \leq x \leq 32.38\)
- \( B \) \(2.62 \leq x \leq 26.55\)
- \( C \) \(-3.2 \leq x \leq 2.62 \) and \(26.55 \leq x \leq 32.38\)
- \( D \) \(0 \leq x \leq 2.62 \) and \(26.55 \leq x \leq 32.38\)
<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.912.A-REI.1.1</td>
<td>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</td>
</tr>
</tbody>
</table>

**Item Types**
- Editing Task Choice – May require choosing the next step in a solution method.
- Equation Editor – May require creating an expression or value.
- GRID – May require dragging and dropping steps, equations, and/or justifications to create a viable argument.
- Hot Text – May require rearranging equations or justifications.
- Multiple Choice – May require identifying expressions, statements, or values.
- Open Response – May require creating a written response.

**Clarifications**
- Students will complete an algebraic proof of solving a linear equation.
- Students will construct a viable argument to justify a solution method.

**Assessment Limit**
- Items will not require the student to recall names of properties from memory.

**Stimulus Attributes**
- Items should be set in a mathematical context.
- Items may use function notation.
- Items should be linear equations in the form of $ax + b = c$, $a(bx + c) = d$, $ax + b = cx + d$, or $a(bx + c) = d(ex + f)$, where $a$, $b$, $c$, $d$, $e$, and $f$ are rational numbers. Equations may be given in forms that are equivalent to these.
- Coefficients may be a rational number or a variable that represents any real number.
- Items should not require more than four procedural steps to reach a solution.

**Response Attributes**
- Items may ask the student to complete steps in a viable argument.
- Items should not ask the student to provide the solution.

**Calculator**
- No
Sample Item

Some of the steps in Raya’s solution to $2.5(6.25x + 0.5) = 11$ are shown.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $2.5(6.25x + 0.5) = 11$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2.</td>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
<td>3. Subtraction property of equality</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
</tbody>
</table>

Drag the correct reason to the box for line 4 of Raya’s solution.

- Closure property
- Distributive property
- Addition property of equality
- Division property of equality
- Symmetric property of equality
**MAFS.912.A-REI.2.4**

Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

**Item Types**

- **Editing Task Choice** – May require choosing steps in a derivation of the quadratic formula.
- **Equation Editor** – May require creating a value or an expression.
- **GRID** – May require dragging and dropping text to complete the derivation of the quadratic formula, or to drag and drop text to complete steps for solving a quadratic equation.
- **Hot Text** – May require rearranging equations.
- **Matching Item** – May require matching quadratic equations with the type of solution (complex or real).
- **Multiple Choice** – May require selecting a value or an expression from a list.
- **Multiselect** – May require selecting multiple values.
- **Open Response** – May require writing an explanation of a step in a solution.

**Clarifications**

- Students will rewrite a quadratic equation in vertex form by completing the square.
- Students will use the vertex form of a quadratic equation to complete steps in the derivation of the quadratic formula.
- Students will solve a simple quadratic equation by inspection or by taking square roots.
- Students will solve a quadratic equation by choosing an appropriate method (i.e., completing the square, the quadratic formula, or factoring).
- Students will validate why taking the square root of both sides when solving a quadratic equation will yield two solutions.
- Students will recognize that the quadratic formula can be used to find complex solutions.
<table>
<thead>
<tr>
<th>Assessment Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>In items that require the student to transform a quadratic equation to vertex form, the coefficient of the linear term must be an even factor of the coefficient of the quadratic term.</td>
</tr>
<tr>
<td>In items that require the student to solve a simple quadratic equation by inspection or by taking square roots, equations should be in the form $ax^2 = c$ or $ax^2 + d = c$, where $a$, $c$, and $d$ are rational numbers and where $c$ is not an integer that is a perfect square and $c - d$ is not an integer that is a perfect square.</td>
</tr>
<tr>
<td>In items that allow the student to choose the method for solving a quadratic equation, equations should be in the form $ax^2 + bx + c = d$, where $a$, $b$, $c$, and $d$ are integers.</td>
</tr>
<tr>
<td>Items may require the student to recognize that a solution is nonreal but should not require the student to find a nonreal solution.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stimulus Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td>The formula must be given in the item for items that can only be solved using the quadratic formula.</td>
</tr>
<tr>
<td>Items should be set in a mathematical context.</td>
</tr>
<tr>
<td>Items may use function notation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response Attributes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items may require the student to complete a missing step in the derivation of the quadratic formula.</td>
</tr>
<tr>
<td>Items may require the student to recognize equivalent solutions to the quadratic equation.</td>
</tr>
<tr>
<td>Responses with square roots should require the student to rewrite the square root so that the radicand has no square factors.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutral</td>
</tr>
<tr>
<td>Sample Item</td>
</tr>
<tr>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Matthew solved the quadratic equation shown.</strong></td>
</tr>
<tr>
<td>(4x^2 - 24x + 7 = 3)</td>
</tr>
<tr>
<td>One of the steps that Matthew used to solve the equation is shown.</td>
</tr>
<tr>
<td>Drag values into the boxes to complete the step and the solution.</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>24</td>
</tr>
<tr>
<td>32</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>148</td>
</tr>
<tr>
<td>154</td>
</tr>
<tr>
<td><strong>Step:</strong> (4(x - \underline{1})^2 = \underline{21})</td>
</tr>
<tr>
<td><strong>Solution:</strong> (x = \underline{4} \pm \sqrt{\underline{21}})</td>
</tr>
</tbody>
</table>
### MAFS.912.A-REI.4.11

**Also assesses MAFS.912.A-REI.4.10**

Also assesses MAFS.912.A-REI.4.10

**Item Types**

- **Equation Editor** – May require creating a value, an equation, or an expression.
- **GRID** – May require identifying points where $f(x) = g(x)$.
- **Hot Text** – May require dragging labels to a graph or dragging and dropping numbers and symbols to complete a solution.
- **Matching Item** – May require choosing ordered pairs that are solutions of a function.
- **Multiple Choice** – May require selecting a value or an expression from a list.
- **Multiselect** – May require selecting multiple values.
- **Open Response** – May require creating a written response.
- **Table Item** – May require completing missing cells in a table.

**Clarifications**

- Students will find a solution or an approximate solution for $f(x) = g(x)$ using a graph.
- Students will find a solution or an approximate solution for $f(x) = g(x)$ using a table of values.
- Students will find a solution or an approximate solution for $f(x) = g(x)$ using successive approximations that give the solution to a given place value.
- Students will justify why the intersection of two functions is a solution to $f(x) = g(x)$.
- Students will verify if a set of ordered pairs is a solution of a function.

**Assessment Limits**

In items where a function is represented by an equation, the function may be an exponential function with no more than one translation, a linear function, or a quadratic function.
In items where a function is represented by a graph or table, the function may be any continuous function.

<table>
<thead>
<tr>
<th>Stimulus Attributes</th>
<th>Items may be set in a mathematical or real-world context.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Items may use function notation.</td>
</tr>
<tr>
<td></td>
<td>Items must designate the place value accuracy necessary for approximate solutions.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response Attributes</th>
<th>Items may require the student to complete a missing step in an algebraic justification of the solution of $f(x) = g(x)$.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Items may require the student to explain the role of the $x$-coordinate and the $y$-coordinate in the intersection of $f(x) = g(x)$.</td>
</tr>
<tr>
<td></td>
<td>Items may require the student to explain a process.</td>
</tr>
<tr>
<td></td>
<td>Items may require the student to record successive approximations used to find the solution of $f(x) = g(x)$.</td>
</tr>
</tbody>
</table>

| Calculator                           | Neutral                                                              |

**Sample Item**

Cora is using successive approximations to estimate a positive solution to $f(x) = g(x)$, where $f(x) = x^2 + 13$ and $g(x) = 3x + 14$. The table shows her results for different input values of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>26</td>
</tr>
<tr>
<td>3.5</td>
<td>25.25</td>
<td>24.5</td>
</tr>
</tbody>
</table>

Use Cora’s process to find the positive solution, to the nearest tenth, of $f(x) = g(x)$.

![Calculator input field]

**Item Type**: Equation Editor
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- Factor a quadratic expression to reveal the zeros of the function it defines.
- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* \(1.15^5\) *can be rewritten as* \((1.15^{1/12})^{12} \approx (1.012)^{12}\) *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Interpret expressions that represent a quantity in terms of its context.

- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* \(P(1+r)^n\) *as the product of* \(P\) *and a factor not depending on* \(P\).

Use the structure of an expression to identify ways to rewrite it. *For example, see* \(x^4 - y^4\) *as* \((x^2)^2 - (y^2)^2\), *thus recognizing it as a difference of squares that can be factored as* \((x^2 - y^2)(x^2 + y^2)\).

**Item Types**

- **Editing Task Choice** – May require choosing equivalent forms of an expression or an interpretation of a parameter.
- **Equation Editor** – May require creating an equivalent expression or numerical response.
- **GRID** – May require dragging and dropping steps in completing the square of a quadratic expression, or in rewriting an expression using algebraic structure.
- **Hot Text** – May require dragging terms, factors, coefficients, or expressions to complete an equivalent expression or to complete an interpretation.
- **Matching Item** – May require matching equivalent expressions.
- **Multiple Choice** – May require selecting an expression or a value from a set of options.
- **Multiselect** – May require selecting expressions or values from a set of options.
- **Open Response** – May require constructing a written response.

**Clarifications**

Students will use equivalent forms of a quadratic expression to interpret the expression’s terms, factors, zeros, maximum, minimum, coefficients, or parts in terms of the real-world situation the expression represents.
Students will use equivalent forms of an exponential expression to interpret the expression’s terms, factors, coefficients, or parts in terms of the real-world situation the expression represents.

Students will rewrite algebraic expressions in different equivalent forms by recognizing the expression’s structure.

Students will rewrite algebraic expressions in different equivalent forms using factoring techniques (e.g., common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely) or simplifying expressions (e.g., combining like terms, using the distributive property, and other operations with polynomials).

### Assessment Limits

In items that require the student to transform a quadratic equation to vertex form, the coefficient of the linear term must be an even factor of the coefficient of the quadratic term.

For A-SSE.1.1, items should not ask the student to interpret zeros, the vertex, or axis of symmetry when the quadratic expression is in the form $ax^2 + bx + c$ (see F-IF.3.8).

For A-SSE.2.3b and A-SSE.1.1, exponential expressions are limited to simple growth and decay. If the number $e$ is used then its approximate value should be given in the stem.

For A-SSE.2.3a and A-SSE.1.1, quadratic expressions should be univariate.

For A-SSE.2.3b, items should only ask the student to interpret the y-value of the vertex within a real-world context.

For A-SSE.2.3, items should require the student to choose how to rewrite the expression.

In items that require the student to write equivalent expressions by factoring, the given expression may have integral common factors, be a difference of two squares up to a degree of 4, be a quadratic, $ax^2 + bx + c$, where $a > 0$ and $a$, $b$, and $c$ are integers, or be a polynomial of four terms with a leading coefficient of 1 and highest degree of 3.

### Stimulus Attributes

Items assessing A-SSE.2.3 and A-SSE.1.1 must be set in a real-world context.

Items that require an equivalent expression found by factoring may be in a real-world or mathematical context.

Items should contain expressions only.
Response Attributes

- Items may require the student to choose an appropriate level of accuracy.
- Items may require the student to choose and interpret units.
- For A-SSE.1.1 and A-SSE.2.3, items may require the student to apply the basic modeling cycle.

Calculator

Neutral

Sample Item

<table>
<thead>
<tr>
<th>Item Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot Text – Selectable</td>
</tr>
</tbody>
</table>

Sue removes the plug from a trough to drain the water inside. The volume, in gallons, in the trough after it has been unplugged can be modeled by \(4t^2 - 32t + 63\), where \(t\) is time, in minutes.

A. Click on the correct property that will give Sue the amount of time it takes the trough to drain.
   - minimum
   - maximum
   - \(y\)-intercept
   - zero

B. Click on the expression that will reveal the property.
   - \(4(0)^2 - 32(0) + 63\)
   - \((2t - 7)(2t - 9)\)
   - \(4(t - 4)^2 - 1\)
   - \(4(t - 8)^2 + 47\)
<table>
<thead>
<tr>
<th>MAFS.912.F-BF.2.3</th>
<th>Identify the effect on the graph of replacing ( f(x) ) by ( f(x) + k ), ( kf(x) ), ( f(kx) ), and ( f(x + k) ) for specific values of ( k ) (both positive and negative); find the value of ( k ) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <em>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>Item Types</td>
<td>Equation Editor – May require creating a value or an expression.</td>
</tr>
<tr>
<td></td>
<td>GRID – May require plotting points or a transformed function.</td>
</tr>
<tr>
<td></td>
<td>Matching Item – May require matching an equation, a value of ( k ), and an explanation of the effect on a graph.</td>
</tr>
<tr>
<td></td>
<td>Multiple Choice – May require selecting a graph or a table from a list.</td>
</tr>
<tr>
<td></td>
<td>Open Response – May require explaining the effects of a transformation.</td>
</tr>
<tr>
<td></td>
<td>Table Item – May require completing a table of values.</td>
</tr>
<tr>
<td>Clarifications</td>
<td>Students will determine the value of ( k ) when given a graph of the function and its transformation.</td>
</tr>
<tr>
<td></td>
<td>Students will identify differences and similarities between a function and its transformation.</td>
</tr>
<tr>
<td></td>
<td>Students will identify a graph of a function given a graph or a table of a transformation and the type of transformation that is represented.</td>
</tr>
<tr>
<td></td>
<td>Students will graph by applying a given transformation to a function.</td>
</tr>
<tr>
<td></td>
<td>Students will identify ordered pairs of a transformed graph.</td>
</tr>
<tr>
<td></td>
<td>Students will complete a table for a transformed function.</td>
</tr>
<tr>
<td>Assessment Limits</td>
<td>Functions represented algebraically are limited to linear, quadratic, or exponential.</td>
</tr>
<tr>
<td></td>
<td>Functions represented using tables or graphs are not limited to linear, quadratic, or exponential.</td>
</tr>
<tr>
<td></td>
<td>Functions may be represented using tables or graphs.</td>
</tr>
<tr>
<td></td>
<td>Functions may have closed domains.</td>
</tr>
<tr>
<td></td>
<td>Functions may be discontinuous.</td>
</tr>
<tr>
<td></td>
<td>Items should have a single transformation.</td>
</tr>
<tr>
<td>Stimulus Attributes</td>
<td>Items should be given in a mathematical context.</td>
</tr>
<tr>
<td></td>
<td>Items may use function notation.</td>
</tr>
</tbody>
</table>
Items may present a function using an equation, a table of values, or a graph.

**Response Attributes**

- Items may require the student to explain or justify a transformation that has been applied to a function.
- Items may require the student to explain how a graph is affected by a value of $k$.
- Items may require the student to find the value of $k$.
- Items may require the student to complete a table of values.

**Calculator**

Neutral

---

**Sample Item**

The table below shows the values for the function $y = f(x)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>-4</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Complete the table for the function $y = f\left(\frac{1}{5}x\right)$.
<table>
<thead>
<tr>
<th>Item Types</th>
<th>Equation Editor – May require expressing a value, an inequality, an expression, or a function.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GRID – May require mapping a relation, or choosing ordered pairs.</td>
</tr>
<tr>
<td></td>
<td>Hot Text – May require dragging and dropping values or a set of values.</td>
</tr>
<tr>
<td></td>
<td>Matching Item– May require selecting cells in a table that associate a function to its domain, values for inputs, or to choose elements of the domain of a relation.</td>
</tr>
<tr>
<td></td>
<td>Multiple Choice – May require selecting a choice from a set of possible domains.</td>
</tr>
<tr>
<td></td>
<td>Multiselect – May require selecting functions from a set of relations.</td>
</tr>
<tr>
<td></td>
<td>Open Response – May require explaining the relationship of related values, or to interpret within a context.</td>
</tr>
<tr>
<td></td>
<td>Table Item – May require completing a table of values.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Clarifications</th>
<th>Students will evaluate functions that model a real-world context for inputs in the domain.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Students will interpret the domain of a function within the real-world context given.</td>
</tr>
<tr>
<td></td>
<td>Students will interpret statements that use function notation within the real-world context given.</td>
</tr>
<tr>
<td></td>
<td>Students will use the definition of a function to determine if a relationship is a function, given tables, graphs, mapping diagrams, or sets of ordered pairs.</td>
</tr>
<tr>
<td></td>
<td>Students will determine the feasible domain of a function that models a real-world context.</td>
</tr>
</tbody>
</table>
**Assessment Limits**

Items that require the student to determine the domain using equations within a context are limited to exponential functions with one translation, linear functions, or quadratic functions.

For F-IF.1.2, in items that require the student to find a value given a function, the following function types are allowed: quadratic, polynomials whose degrees are no higher than 6, square root, cube root, absolute value, exponential except for base \( e \), and simple rational.

Items may present relations in a variety of formats, including sets of ordered pairs, mapping diagrams, graphs, and input/output models.

In items requiring the student to find the domain from graphs, relationships may be on a closed or open interval.

In items requiring the student to find domain from graphs, relationships may be discontinuous.

Items may not require the student to use or know interval notation.

**Stimulus Attributes**

For F-IF.1.1, items may be set in a real-world or mathematical context.

For F-IF.1.2, items that require the student to evaluate may be written in a mathematical or real-world context. Items that require the student to interpret must be set in a real-world context.

For F-IF.2.5, items must be set in a real-world context.

Items must use function notation.

**Response Attributes**

For F-IF.2.5, items may require the student to apply the basic modeling cycle.

Items may require the student to choose an appropriate level of accuracy.

Items may require the student to choose and interpret the scale in a graph.

Items may require the student to choose and interpret units.

Items may require the student to write domains using inequalities.

**Calculator**

Neutral
The points on the graph show the population data, in millions, of the state of Florida for each decade from 1900 to 2000. The data are modeled by the function $P(x) = 506975(1.43)^x$, shown on the graph.

What is the domain of the graph of $P(x)$ that is shown?

- A. $x \geq 0$
- B. $1900 \leq x \leq 2000$
- C. all whole numbers
- D. $0 \leq x \leq 10$
<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAFS.912.F-IF.2.4</td>
<td>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</td>
</tr>
<tr>
<td>Also assesses MAFS.912.F-IF.3.9</td>
<td>Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</td>
</tr>
</tbody>
</table>

**Item Types**

- **Equation Editor** – May require expressing a value, expression, or equation.
- **GRID** – May require plotting points on a coordinate plane, graphing a function, or matching and/or selecting key features as verbal descriptions to points on the graph.
- **Hot Text** – May require selecting a key feature or region on a graph.
- **Multiple Choice** – May require selecting a choice from a set of possible choices.
- **Open Response** – May require explaining the meaning of key features or the comparison of two functions.

**Clarifications**

- Students will determine and relate the key features of a function within a real-world context by examining the function’s table.
- Students will determine and relate the key features of a function within a real-world context by examining the function’s graph.
- Students will use a given verbal description of the relationship between two quantities to label key features of a graph of a function that model the relationship.
- Students will differentiate between different types of functions using a variety of descriptors (e.g., graphically, verbally, numerically, and algebraically).
- Students will compare and contrast properties of two functions using a variety of function representations (e.g., algebraic, graphic, numeric in tables, or verbal descriptions).

**Assessment Limits**

- Functions represented algebraically are limited to linear, quadratic, or exponential.
- Functions may be represented using tables, graphs or verbally. Functions represented using these representations are not limited to linear, quadratic or exponential.
Functions may have closed domains.

Functions may be discontinuous.

Items may not require the student to use or know interval notation.

Key features include x-intercepts, y-intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

**Stimulus Attributes**

For F-IF.2.4, items should be set in a real-world context.

For F-IF.3.9, items may be set in a real-world or mathematical context.

Items may use verbal descriptions of functions.

Items may use function notation.

**Response Attributes**

For F-IF.2.4, items may require the student to apply the basic modeling cycle.

Items may require the student to write intervals using inequalities.

Items may require the student to choose an appropriate level of accuracy.

Items may require the student to choose and interpret the scale in a graph.

Items may require the student to choose and interpret units.

**Calculator**

No
Sample Item

Kim is driving from Miami to Key West. The graph shows her distance from Key West.

During what interval is Kim driving the fastest? Drag numbers to the boxes to complete the inequality.

\[
\leq x \leq \phantom{0}
\]
### MAFS.912.F-IF.2.6

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

#### Item Types

- **Equation Editor** – May require creating rate of change as a numeric value.
- **Hot Text** – May require dragging and dropping phrases or values.
- **Matching Item** – May require matching a value with an interpretation.
- **Multiple Choice** – May require selecting a statement about the rate of a data display, an interpretation, or context.
- **Multiselect** – May require selecting multiple statements about the rate of change and/or the constant term in a given context.
- **Open Response** – May require explaining the rate of change or y-intercept in context.

#### Clarifications

- Students will calculate the average rate of change of a continuous function that is represented algebraically, in a table of values, on a graph, or as a set of data.
- Students will interpret the average rate of change of a continuous function that is represented algebraically, in a table of values, on a graph, or as a set of data with a real-world context.
- Students will interpret the y-intercept of a linear model that represents a set of data with a real-world context.

#### Assessment Limits

- Items requiring the student to calculate the rate of change will give a specified interval that is both continuous and differentiable.
- Items should not require the student to find an equation of a line.
- Items assessing S-ID.3.7 should include data sets. Data sets must contain at least six data pairs. The linear function given in the item should be the regression equation.
- For items assessing S-ID.3.7, the rate of change and the y-intercept should have a value with at least a hundredths place value.
Items assessing F-IF.2.6 should not be linear.

| Stimulus Attributes | Items may require the student to apply the basic modeling cycle.  
| Items should be set in a real-world context.  
| Items may use function notation.  
| Items may require the student to choose and interpret variables. |

| Response Attributes | Items may require the student to choose an appropriate level of accuracy.  
| Items may require the student to choose and interpret the scale in a graph.  
| Items may require the student to choose and interpret units. |

| Calculator | Neutral |

**Sample Item**

The graph shows the number of acres, in millions, of farmland in the United States from 1975 to 2008.

**U.S. Farmland Area**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Acres (In millions)</td>
<td>1,080</td>
<td>1,060</td>
<td>1,040</td>
<td>1,020</td>
<td>1,000</td>
<td>980</td>
<td>960</td>
<td>940</td>
</tr>
</tbody>
</table>

Which statement describes the average rate of change of the graph?

- A. The number of acres of farmland in the United States decreases by 0.21 million each year.
- B. The amount of farmland in the United States decreases by 4.8 million acres each year.
- C. The time it takes the farmland in the United States to decrease by 160 acres is 33 years.
- D. Every 5 years, the amount of farmland in the United States decreases by 20 acres.
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as \( y = (1.02)^t \), \( y = (0.97)^t \), \( y = (1.01)^{-t} \), and \( y = (1.2)^{10t} \) and classify them as representing exponential growth or decay.

Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.

Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases.

- Graph linear and quadratic functions and show intercepts, maxima, and minima.
- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior.
- Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior.
- Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude and using phase shift.

<table>
<thead>
<tr>
<th>Item Types</th>
<th>Equation Editor – May require creating a value, an expression, or an equation.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GRID – May require plotting points, key features, or an equation on a graph; identifying key features; or selecting key features on a graph.</td>
</tr>
<tr>
<td></td>
<td>Hot Text – May require selecting key features on a graph.</td>
</tr>
<tr>
<td></td>
<td>Multiple Choice – May require selecting from a list.</td>
</tr>
<tr>
<td></td>
<td>Multiselect – May require selecting multiple responses.</td>
</tr>
<tr>
<td></td>
<td>Open Response – May require explaining and interpreting a function.</td>
</tr>
</tbody>
</table>

Students will identify zeros, extreme values, and symmetry of a quadratic function written symbolically.
Students will classify the exponential function as exponential growth or decay by examining the base, and students will give the rate of growth or decay.

Students will use the properties of exponents to interpret exponential expressions in a real-world context.

Students will write an exponential function defined by an expression in different but equivalent forms to reveal and explain different properties of the function, and students will determine which form of the function is the most appropriate for interpretation for a real-world context.

Students will find the zeros of a polynomial function when the polynomial is in factored form.

Students will create a rough graph of a polynomial function in factored form by examining the zeros of the function.

Students will use the x-intercepts of a polynomial function and end behavior to graph the function.

Students will identify the x- and y-intercepts and the slope of the graph of a linear function.

Students will identify zeros, extreme values, and symmetry of the graph of a quadratic function.

Students will identify intercepts and end behavior for an exponential function.

Students will graph a linear function using key features.

Students will graph a quadratic function using key features.

Students will graph an exponential function using key features.

Students will identify and interpret key features of a graph within the real-world context that the function represents.

**Assessment Limits**

For A-APR.2.3, the leading coefficient should be an integer and the polynomial’s degree is restricted to 3 or 4. The polynomial function should not have a zero with multiplicity. The polynomial should be given in factored form.

For F-IF.3.8a, items that require the student to transform a quadratic equation to vertex form, the coefficient of the linear term must be an even factor of the coefficient of the quadratic form.

For F-IF.3.7e and F-IF.3.8b, exponential functions are limited to simple...
exponential growth and decay functions and to exponential functions with one translation. Base e should not be used.

For F-IF.3.8, items may specify a required form using an equation or using common terminology such as standard form.

In items that require the student to interpret the vertex or a zero of a quadratic function within a real-world context, the student should interpret both the $x$-value and the $y$-value.

For F-IF.3.7a, quadratic functions that are given in the form $y = ax^2 + bx + c$, $a$, $b$, and $c$ must be integers. Quadratic functions given in vertex form $y = a(x - h)^2 + k$, $a$, $h$, and $k$ must be integers. Quadratic functions given in other forms should be able to be rewritten and adhere to one of the two previous forms.

<table>
<thead>
<tr>
<th>Stimulus Attributes</th>
<th>Items may require the student to identify a correct graph.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Items may be set in a mathematical or real-world context.</td>
</tr>
<tr>
<td></td>
<td>Items may use function notation.</td>
</tr>
<tr>
<td></td>
<td>Items should not require the student to complete a sign chart for a polynomial.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response Attributes</th>
<th>For F-IF.3.7, items may require the student to apply the basic modeling cycle.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Items may require the student to choose an appropriate level of accuracy.</td>
</tr>
<tr>
<td></td>
<td>Items may require the student to choose and interpret the scale in a graph.</td>
</tr>
<tr>
<td></td>
<td>Items may require the student to choose and interpret units.</td>
</tr>
<tr>
<td></td>
<td>Responses with square roots should require the student to rewrite the square root so that the radicand has no square factors.</td>
</tr>
</tbody>
</table>

| Calculator          | Neutral |
A bird drops a stick from the top of Miami Tower. The height of the stick after $x$ seconds is given by $f(x) = 625 - 16x^2$. What is the maximum value of $f(x)$?

Select all the correct interpretations of the coordinates of the point at the maximum of the function $f(x)$.

- the time it takes the stick to hit the ground
- the time when the stick is at its highest point
- the height of the stick when it is dropped from Miami Tower
- the distance of the stick from Miami Tower when it hits the ground
- the time when the stick is dropped from the top of the Miami Tower
| MAFS.912.F-LE.1.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions.  
| | a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.  
| | b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.  
| | c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.  
| Also assesses MAFS.912.F-LE.2.5 | Interpret the parameters in a linear or exponential function in terms of a context.  
| Item Types | Editing Task Choice – May require choosing a model, a parameter, and/or an interpretation.  
| | Equation Editor – May require creating a value or an expression.  
| | GRID – May require dragging and dropping expressions or statements to a graph.  
| | Hot Text – May require dragging and dropping justifications or interpretations.  
| | Matching Item – May require matching parameters with interpretations.  
| | Multiple Choice – May require selecting an interpretation from a list.  
| | Multiselect – May require selecting multiple values.  
| | Open Response – May require analyzing the growth of a function or explaining parameters of a function.  
| Clarifications | Students will determine whether the real-world context may be represented by a linear function or an exponential function and give the constant rate or the rate of growth or decay.  
| | Students will choose an explanation as to why a context may be modeled by a linear function or an exponential function.  
| | Students will interpret the rate of change and intercepts of a linear function when given an equation that models a real-world context.  
| | Students will interpret the x-intercept, y-intercept, and/or rate of growth or decay of an exponential function given in a real-world context.  
| Assessment Limit | Exponential functions should be in the form $a(b)^x + k$.  
| Stimulus Attributes | Items should be set in a real-world context.  
| | Items may use function notation.  

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### Response Attributes

- Items may require the student to apply the basic modeling cycle.
- Items may require the student to choose a parameter that is described within the real-world context.
- Items may require the student to choose an appropriate level of accuracy.
- Items may require the student to choose and interpret the scale in a graph.
- Items may require the student to choose and interpret units.

### Calculator

No

---

### Sample Item

The graph shows $T$, the temperature of water, in degrees Celsius, in a test tube after $m$ minutes of an experiment.

Drag a label to each box to correctly identify the type of rate of change between temperature and time on each part of the graph.
### MAFS.912.F-LE.1.2

Also assesses MAFS.912.F-BF.1.1

Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).

Write a function that describes a relationship between two quantities.

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **b.** Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
- **c.** Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

### Item Types

<table>
<thead>
<tr>
<th>Item Types</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Editing Task Choice</td>
<td>May require choosing an expression, function, or definition of a variable.</td>
</tr>
<tr>
<td>Equation Editor</td>
<td>May require creating a value, creating an expression, creating a function, or showing steps for a calculation.</td>
</tr>
<tr>
<td>GRID</td>
<td>May require ordering of steps for a calculation from a context.</td>
</tr>
<tr>
<td>Hot Text</td>
<td>May require dragging and dropping values or expressions to construct a function.</td>
</tr>
<tr>
<td>Multiple Choice</td>
<td>May require selecting a choice from a set of possible choices.</td>
</tr>
<tr>
<td>Multiselect</td>
<td>May require choosing equivalent functions.</td>
</tr>
<tr>
<td>Open Response</td>
<td>May require explaining and interpreting a resulting function.</td>
</tr>
<tr>
<td>Table Item</td>
<td>May require completing missing cells in a table.</td>
</tr>
</tbody>
</table>

### Clarifications

Students will write a linear function, an arithmetic sequence, an exponential function, or a geometric sequence when given a graph that models a real-world context.

Students will write a linear function, an arithmetic sequence, an exponential function, or a geometric sequence when given a verbal description of a real-world context.
Students will write a linear function, an arithmetic sequence, an exponential function, or a geometric sequence when given a table of values or a set of ordered pairs that model a real-world context.

Students will write an explicit function, define a recursive process, or complete a table of calculations that can be used to mathematically define a real-world context.

Students will write a function that combines functions using arithmetic operations and relate the result to the context of the problem.

Students will write a function to model a real-world context by composing functions and the information within the context.

Students will write a recursive definition for a sequence that is presented as a sequence, a graph, or a table.

**Assessment Limits**

In items where the student must write a function using arithmetic operations or by composing functions, the student should have to generate the new function only.

In items where the student constructs an exponential function, a geometric sequence, or a recursive definition from input-output pairs, at least two sets of pairs must have consecutive inputs.

In items that require the student to construct arithmetic or geometric sequences, the real-world context should be discrete.

In items that require the student to construct a linear or exponential function, the real-world context should be continuous.

**Stimulus Attributes**

Items should be set in a real-world context.

Items may use function notation.

In items where the student builds a function using arithmetic operations or by composition, the functions may be given using verbal descriptions, function notation or as equations.

**Response Attributes**

For F-BF.1.1b and c, the student may be asked to find a value.

For F-LE.1.2 and F-BF.1.1, items may require the student to apply the basic modeling cycle.

In items where the student writes a recursive formula, the student may be expected to give both parts of the formula.

The student may be required to determine equivalent recursive formulas or functions.

Items may require the student to choose an appropriate level of accuracy.
Items may require the student to choose and interpret the scale in a graph.
Items may require the student to choose and interpret units.

| Calculator | Neutral |

Sample Item

**Item Type**

**Equation Editor**

Chantel drew a picture of her dog on a piece of paper that is 12 centimeters long. She used a copy machine to enlarge her drawing. She used the 115% setting to make each new copy. She then used each new copy to generate the next copy, using the same copier setting.

Enter a recursive formula that will give the length of each new copy.

\[
a_1 = \_
\]

\[
a_n = \_
\]
<table>
<thead>
<tr>
<th>MAFS.912.F-LE.1.3</th>
<th>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</th>
</tr>
</thead>
</table>
| **Item Types**    | Editing Task Choice – May require choosing a function and/or a justification.  
Equation Editor – May require creating a value or an expression.  
GRID – May require selecting a part of a graph or table.  
Hot Text – May require rearranging equations.  
Multiple Choice – May require selecting a value or an expression from a list.  
Multiselect – May require selecting multiple values.  
Open Response – May require explaining what happens to a function for large values of $x$ or explaining a comparison. |
| **Clarifications**| Students will compare a linear function and an exponential function given in real-world context by interpreting the functions’ graphs.  
Students will compare a linear function and an exponential function given in a real-world context through tables.  
Students will compare a quadratic function and an exponential function given in real-world context by interpreting the functions’ graphs.  
Students will compare a quadratic function and an exponential function given in a real-world context through tables. |
| **Assessment Limits** | Exponential functions represented in graphs or tables should be able to be written in the form $a(b)^x + k$.  
For exponential relationships, tables or graphs must contain at least one pair of consecutive values. |
| **Stimulus Attributes** | Items should give a graph or a table.  
Items should be given in a real-world context.  
Items may use function notation. |
| **Response Attributes** | Items may require the student to apply the basic modeling cycle.  
Items may require the student to choose an appropriate level of accuracy.  
Items may require the student to choose and interpret the scale in a graph.  
Items may require the student to choose and interpret units. |
| **Calculator** | No |
The function \( f(x) \) models the value of goods that are imported into the United States, where \( x \) is the number of years since 1990. The function \( g(x) \) models the value of goods that are exported from the United States.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>$8,859,296.92</td>
<td>$6,295,111.00</td>
</tr>
<tr>
<td>45</td>
<td>$10,308,975.90</td>
<td>$8,476,064.00</td>
</tr>
<tr>
<td>50</td>
<td>$11,833,485.40</td>
<td>$11,412,611.00</td>
</tr>
<tr>
<td>51</td>
<td>$12,147,367.00</td>
<td>$12,112,204.00</td>
</tr>
<tr>
<td>52</td>
<td>$12,464,241.80</td>
<td>$12,854,683.00</td>
</tr>
<tr>
<td>55</td>
<td>$13,432,825.40</td>
<td>$15,366,531.00</td>
</tr>
<tr>
<td>60</td>
<td>$15,106,996.00</td>
<td>$20,690,294.00</td>
</tr>
</tbody>
</table>

If \( f(x) \) and \( g(x) \) continue to model the importing and exporting of goods, then sometime in 2041, which is 51 years after 1990, \( f(x) = g(x) \).

Determine which function is exponential. Use the table of values to justify your choice.

Type your answer in the space provided. Be sure to include your function choice.
### MAFS.912.N-RN.1.2

*Also assesses MAFS.912.N-RN.1.1*

Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $(5^{\frac{1}{3}})^3 = 5(\frac{1}{3})^3$ to hold, so $(5^{\frac{1}{3}})^3$ must equal 5.*

Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

### Item Types

**Editing Task Choice** – May require choosing a value, an expression, or a statement.

**Equation Editor** – May require creating a value or an expression.

**GRID** – May require identifying parts of an algebraic proof.

**Hot Text** – May require dragging and dropping values, expressions, or explanations.

**Matching Item** – May require matching equivalent expressions.

**Multiple Choice** – May require selecting a value or an expression from a list.

**Multiselect** – May require selecting multiple values.

**Open Response** – May require explaining why two rational exponent expressions are equivalent or why two expressions are equivalent.

### Clarifications

Students will use the properties of exponents to rewrite a radical expression as an expression with a rational exponent.

Students will use the properties of exponents to rewrite an expression with a rational exponent as a radical expression.

Students will apply the properties of operations of integer exponents to expressions with rational exponents.

Students will apply the properties of operations of integer exponents to radical expressions.
Students will write algebraic proofs that show that a sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

**Assessment Limits**  
Expressions should contain no more than three variables.

For N-RN.1.2, items should not require the student to do more than two operations.

**Stimulus Attribute**  
Items should be set in a mathematical context.

**Response Attributes**  
Items may require the student to complete an algebraic proof.

Items may require the student to determine equivalent expressions or equations.

Responses with square roots should require the student to rewrite the square root so that the radicand has no square factors.

**Calculator**  
No

<table>
<thead>
<tr>
<th>Sample Item</th>
<th>Item Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jeremy shows that $\sqrt{9} = 9^{\frac{1}{2}}$. Part of his work is shown.</td>
<td>Multiple Choice</td>
</tr>
<tr>
<td>$\sqrt{9} = 3 = 3^1 = 3^{\frac{1}{2}} + 3^{\frac{1}{2}} = ___ = 9^{\frac{1}{2}}$</td>
<td></td>
</tr>
<tr>
<td>Which expression or equation can be placed in the blank to correctly complete Jeremy's work?</td>
<td></td>
</tr>
<tr>
<td>A  $(3^2)^{\frac{1}{2}}$</td>
<td></td>
</tr>
<tr>
<td>B  $3^{\frac{1}{2}} + 3^{\frac{1}{2}}$</td>
<td></td>
</tr>
<tr>
<td>C  $3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = (3 \cdot 3)^{\frac{1}{2}}$</td>
<td></td>
</tr>
<tr>
<td>D  $3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} = (3 \cdot 3)^{\frac{1}{2} + \frac{1}{2}}$</td>
<td></td>
</tr>
</tbody>
</table>
| Item Types | GRID – May require interacting with data displays (i.e., creating a dot plot by clicking on a number in a number line to generate a set number of points), or labeling components of a graph (i.e., median lower quartile, upper quartile, and/or outlier).

Hot Text – May require labeling components of a set of data (i.e., median, lower quartile, upper quartile, and/or outlier).

Multiple Choice – May require selecting a graph from a set.

Multiselect – May require selecting various representations of the same data.

Open Response – May require critiquing the usage of certain displays and explaining general factors that contribute to selecting the most appropriate data display. |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Clarity</td>
<td>Students will represent data using a dot plot, a histogram, or a box plot.</td>
</tr>
<tr>
<td>Assessment Limits</td>
<td>None</td>
</tr>
<tr>
<td>Stimulus Attribute</td>
<td>Items should use real-world data and be set in a real-world context.</td>
</tr>
</tbody>
</table>
| Response Attributes | Items may require the student to apply the basic modeling cycle.

Items may require the student to choose an appropriate level of accuracy.

Items may require the student to choose and interpret the scale in a graph.

Items may require the student to choose and interpret units. |
| Calculator | Neutral |
Max collected data on the height of each of his 20 classmates. The box plot shown represents his data.

Click above the number line to complete the dot plot that could also represent these data.
### Item Specifications

#### MAFS.912.S-ID.1.2

Also assesses MAFS.912.S-ID.1.3

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

<table>
<thead>
<tr>
<th>Item Types</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Editing Task Choice</td>
<td>May require choosing a correct interpretation.</td>
</tr>
<tr>
<td>Equation Editor</td>
<td>May require providing a numeric value (mean, median, and/or interquartile range).</td>
</tr>
<tr>
<td>GRID</td>
<td>May require plotting points on a number line (i.e., indicate quartiles of a box plot or median and mean of a spread).</td>
</tr>
<tr>
<td>Hot Text</td>
<td>May require interacting with a data spread (i.e., indication of standard deviations, percentages of values in the spread).</td>
</tr>
<tr>
<td>Matching Item</td>
<td>May require matching data pieces and their effect on the shape, center, spread, interquartile range, or standard deviation.</td>
</tr>
<tr>
<td>Multiple Choice</td>
<td>May require selecting a statement or graph from a set or selecting a graphical representation of a data set that is approximately normally distributed.</td>
</tr>
<tr>
<td>Multiselect</td>
<td>May require choosing similarities between data sets.</td>
</tr>
<tr>
<td>Open Response</td>
<td>May require explaining the differences/similarities between two data sets.</td>
</tr>
</tbody>
</table>

**Clarifications**

Students will identify similarities and differences in shape, center, and spread when given two or more data sets.

Students will predict the effect that an outlier will have on the shape, center, and spread of a data set.

Students will interpret similarities and differences in shape, center, and spread when given two or more data sets within the real-world context given.

Students will use their understanding of normal distribution and the empirical rule to answer questions about data sets.

**Assessment Limits**

Items may require the student to calculate mean, median, and interquartile range for the purpose of identifying similarities and differences.

Items should not require the student to calculate the standard deviation.

Items should not require the student to fit normal curves to data.
Data distributions should be approximately normal.
Data sets should be real-world and quantitative.

<table>
<thead>
<tr>
<th>Stimulus Attributes</th>
<th>In items that require standard deviation, the value should be given in the stem.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Items should use real-world data and be set in a real-world context.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Response Attributes</th>
<th>Items may require the student to apply the basic modeling cycle.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Items may require the student to choose an appropriate level of accuracy.</td>
</tr>
<tr>
<td></td>
<td>Items may require the student to choose and interpret the scale in a graph.</td>
</tr>
<tr>
<td></td>
<td>Items may require the student to choose and interpret units.</td>
</tr>
</tbody>
</table>

| Calculator | Neutral |

---

### Sample Item

<table>
<thead>
<tr>
<th>Florida</th>
<th>Texas</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>County</strong></td>
<td><strong>Population</strong></td>
</tr>
<tr>
<td>Smallest</td>
<td>8,349</td>
</tr>
<tr>
<td>First Quartile</td>
<td>27,013</td>
</tr>
<tr>
<td>Median</td>
<td>107,056</td>
</tr>
<tr>
<td>Third Quartile</td>
<td>337,362</td>
</tr>
<tr>
<td>Largest</td>
<td>2,617,176</td>
</tr>
</tbody>
</table>

Florida has 67 counties and Texas has 254 counties.

- The mean population for the state of Florida by county is 291,834 with a standard deviation of 467,012.03, and the median is 107,056.
- The mean population for the state of Texas by county is 104,127 with a standard deviation of 374,012.2261, and the median is 18,293.

Some of the data for both states is shown.

A business moves its corporate location from Texas to Florida. As a result of the move, 8,193 people move from the largest Texas county to the smallest Florida county, in terms of population.

Select all the population statistics that will be affected with this population change.

<table>
<thead>
<tr>
<th>Increases</th>
<th>Decreases</th>
<th>Stays the Same</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interquartile Range of Florida</td>
<td>☐</td>
<td>☑</td>
</tr>
<tr>
<td>Mean of Texas</td>
<td>☑</td>
<td>☐</td>
</tr>
<tr>
<td>Median of Florida</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Standard Deviation of Texas</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Standard</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>MAFS.912.S-ID.2.5</td>
<td>Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</td>
<td></td>
</tr>
</tbody>
</table>

**Item Types**

- **Editing Task Choice** – May require choosing a correct interpretation.
- **Equation Editor** – May require providing a numeric value.
- **GRID** – May require constructing a frequency table.
- **Hot Text** – May require identifying marginal frequencies on a frequency table or constructing a frequency table.
- **Matching Item** – May require matching relative frequencies with categorical data.
- **Multiple Choice** – May require selecting a contingency table or selecting a numeric value.
- **Multiselect** – May require choosing relative frequencies, associations, and/or trends for a two-way frequency table.
- **Open Response** – May require interpreting relative frequencies in the context of the data.
- **Table Item** – May require completing a table.

**Clarifications**

- Students will create or complete a two-way frequency table to summarize categorical data.
- Students will determine if associations/trends are appropriate for the data.
- Students will interpret data displayed in a two-way frequency table.
- Students will calculate joint, marginal, and conditional relative frequencies.

**Assessment Limit**

In data with only two categorical variables, items should require the student to determine relative frequencies and use the frequencies to complete the table or to answer questions.

**Stimulus Attribute**

Items should use real-world data and be set in a real-world context.

**Response Attributes**

- Items may require the student to apply the basic modeling cycle.
- Items may require the student to choose an appropriate level of accuracy.
- Items may require the student to choose and interpret units.

**Calculator**

Yes
A high school drama teacher organizes a musical production. He wants to record the number of students involved in each part of the production. He uses a two-way table to display the data.

The drama teacher knows that approximately 55% more girls participate in the production as actors than as stage crew members.

Complete the two-way table to show a possible breakdown of students.

<table>
<thead>
<tr>
<th></th>
<th>Actors</th>
<th>Musicians</th>
<th>Stage Crew</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>17</td>
<td></td>
<td></td>
<td>53</td>
</tr>
<tr>
<td>Girls</td>
<td></td>
<td>22</td>
<td></td>
<td>73</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>43</td>
<td>126</td>
</tr>
</tbody>
</table>
| MAFS.912.S-ID.2.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.  
  a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.  
  b. Informally assess the fit of a function by plotting and analyzing residuals.  
  c. Fit a linear function for a scatter plot that suggests a linear association.  

Also assesses
MAFS.912.S-ID.3.8

Also assesses
MAFS.912.S-ID.3.9

Compute (using technology) and interpret the correlation coefficient of a linear fit.

Distinguish between correlation and causation.

**Item Types**

- **Editing Task Choice** – May require choosing a correct interpretation.
- **Equation Editor** – May require creating an equation or providing a residual value.
- **GRID** – May require constructing a scatter plot, plotting residual values, or graphing a line of best fit.
- **Hot Text** – May require labeling parts of a graph.
- **Matching Item** – May require matching scatter plots with functions.
- **Multiple Choice** – May require selecting a linear equation or graph from a set, selecting a scatterplot graph that can or cannot fit a function, selecting a numeric value or a graph from a set, or selecting a statement describing the data given in reference to the correlation.
- **Multiselect** – May require selecting multiple scatterplot graphs that can or cannot fit a function or selecting statements describing the data given in reference to the correlation and/or causation.
- **Open Response** – May require explaining why certain data cannot fit into a best fit line or identifying flaws in a data display, summarizing an interpretation of a graph (i.e., correlation) or explaining why a relationship is not causal.

**Clarifications**

Students will represent data on a scatter plot.

Students will identify a linear function, a quadratic function, or an exponential function that was found using regression.

Students will use a regression equation to solve problems in the context of the data.
Students will calculate residuals.

Students will create a residual plot and determine whether a function is an appropriate fit for the data.

Students will determine the fit of a function by analyzing the correlation coefficient.

Students will distinguish between situations where correlation does not imply causation.

Students will distinguish variables that are correlated because one is the cause of another.

<table>
<thead>
<tr>
<th>Assessment Limit</th>
<th>In items that require the student to interpret or use the correlation coefficient, the value of the correlation coefficient must be given in the stem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulus Attribute</td>
<td>Items should use real-world data and be set in a real-world context.</td>
</tr>
<tr>
<td>Response Attributes</td>
<td>Items may require the student to apply the basic modeling cycle.</td>
</tr>
<tr>
<td></td>
<td>Items may require the student to choose an appropriate level of accuracy.</td>
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<tr>
<td></td>
<td>Items may require the student to choose and interpret the scale in a graph.</td>
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<tr>
<td></td>
<td>Items may require the student to choose and interpret units.</td>
</tr>
<tr>
<td>Calculator</td>
<td>Neutral</td>
</tr>
</tbody>
</table>
A company creates the equation $y = 11.26x - 76.1$ to model the relationship between the number of pages in its catalog and the number of orders, in thousands, that were received.

To determine how well the equation models the relationship, the company plots the residuals as shown.

**Residual Plot**

Why is the equation not a good model for the relationship?

Type your answer in the space provided.
Algebra 1 Reference Sheet

Customary Conversions
1 foot = 12 inches
1 yard = 3 feet
1 mile = 5,280 feet
1 mile = 1,760 yards

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

1 pound = 16 ounces
1 ton = 2,000 pounds

Metric Conversions
1 meter = 100 centimeters
1 meter = 1000 millimeters
1 kilometer = 1000 meters

1 liter = 1000 milliliters

1 gram = 1000 milligrams
1 kilogram = 1000 grams

Time Conversions
1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 year = 365 days
1 year = 52 weeks